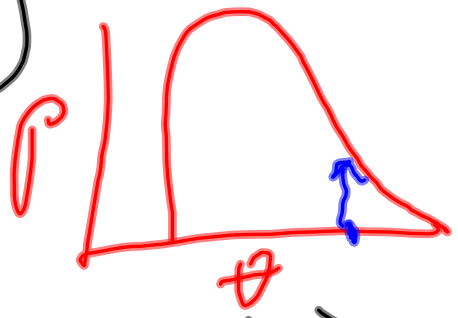
Bayesian Theory

Baye's Theorem



$$p(\theta/x) \propto \pi(\theta)\mathcal{L}(x/\theta)$$

$$p(\theta/x) = \frac{\pi(\theta)\mathcal{L}(x/\theta)}{\int_{\theta} \pi(\theta)\mathcal{L}(x/\theta)\,d\theta}$$

① Prior may prove uninfluential in a given case, if the data are "overwhelming" (but not if the likelihood shows non-identifiability).

② If the prior is influential, it needs to be empirically "true".

③ Every unknown or "uncertain" element must be a parameter.

④ Every parameter must have a prior (i.e. appear in the joint prior)

PVA Retrospective Analysis with Measurement Error

Imagine that retrospective analysis is based on census trajectory where there is census error

Consider Brownian Motion

PVA:

Environmental variation in λ has a mean and a std.

(ignore demographic stochasticity)

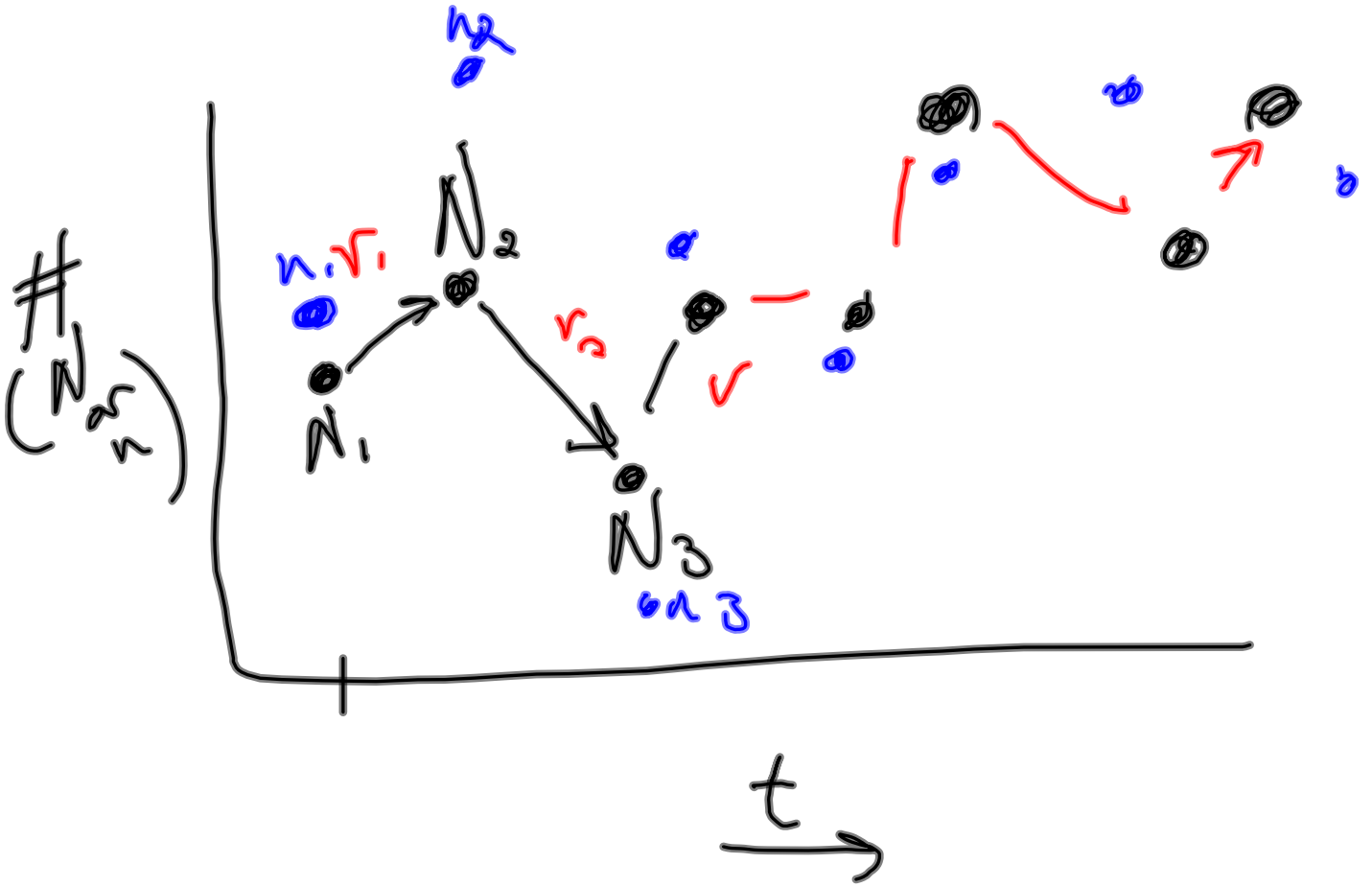
r $(= \ln \lambda)$

μ_r (mean of distribution of environmental variability - normal)

σ_r

N true population in log space

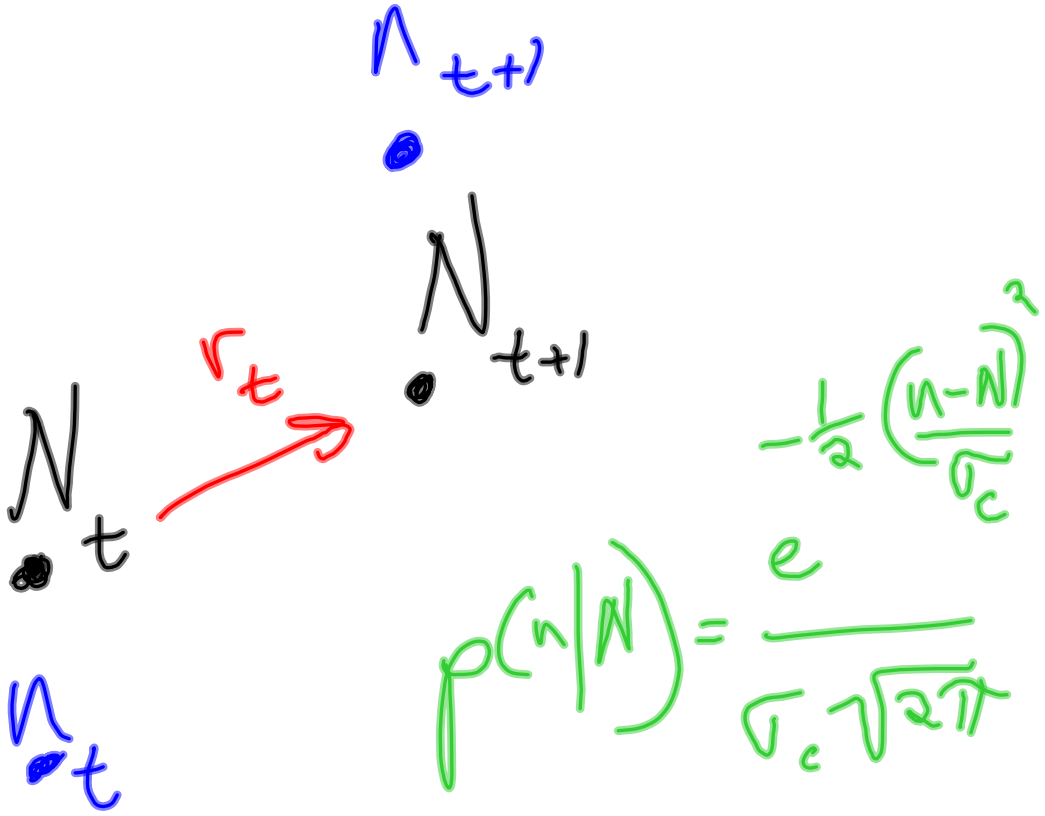
n observed census estimate in log space



state descriptor

state equation

observed


n_{t+1}
N_{t+1}
r_t
N_t
n_t
$-\frac{1}{2}\left(\frac{n-N}{\sigma_c}\right)^2$
$p(n|N) = \frac{e}{\sigma_c\sqrt{2\pi}}$
$\mathcal{L}(n|N)$
"Census error distribution"

State equation

$$p(N_{t+1} \mid N_t) = \frac{e^{-\frac{1}{2}\left(\frac{N_{t+1} - N_t - \mu_r}{\sigma_r}\right)^2}}{\sigma_r \sqrt{2\pi}}$$

Components of the prior

Other components of prior

$\pi(\mu_r)$ "comparative, empirical, literature"

$\pi(\sigma_r)$ "habitat"

$\pi(N)$

Log Joint Prior

$$\ln(\pi(\mu)) + \ln(\pi(\sigma^2)) + \sum_{j=1}^{k} \ln(\pi(N_j)) + \sum_{j=1}^{k-1} -\frac{1}{2}\left(\frac{N_{j+1} - N_j - \mu_a}{\sigma_a}\right)^2 - (k-1)\ln(\sigma_a\sqrt{2\pi})$$

Log Joint Likelihood

$$\sum_{j=1}^{k} -\frac{1}{2}\left(\frac{\eta_j - N_j}{\sigma_c}\right)^2 - k \ln\left(\sigma_c \sqrt{2\pi}\right)$$

Feed Log Joint Prior
+ Log Joint Likelihood
to MCMC

See if marginal posterior
for μ_r and Γ_r differ
from their priors
(depend on k and σ_c)

Building in Density-Dependence

$$m_r \overset{?}{=} f(N)$$

$$m_r = a + bN$$

add a parameter

need priors for π_a π_b