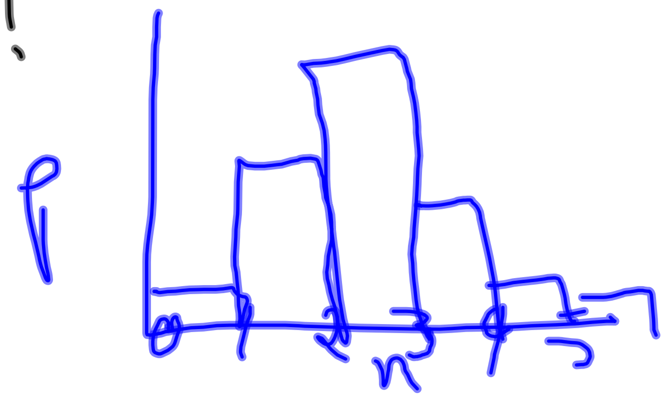
$$p(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$



0	0	3	1	0					

if $\underline{\lambda}$ then with a very large sample size $\bar{n} = \lambda$

Stochastic process

process
λ $\longrightarrow$ outcomes
"data"

Inference

λ? $\longleftarrow$ "data"

Consider:
* "area" of grid cell-well is t
* λ is "per unit"
* n_i: count in cell-well i

Poisson

$$p(n_i \mid \lambda, t) = \frac{e^{-\lambda t} (\lambda t)^{n_i}}{n_i!}$$

$$E(n_i) = \lambda t$$

$$Var(n_i) = \lambda t$$

Observe: $n_i = N$

Intuitive estimator:

N approximates $E(n) = \lambda t$

$$N = \hat{\lambda} t$$

$$\hat{\lambda} = \frac{N}{t}$$

"unbiased": $E(\hat{\lambda}) = \lambda$

$$E(\hat{\lambda}) = E\left(\frac{N}{t}\right) = \frac{1}{t} E(N) = \frac{\lambda t}{t} = \lambda$$

$Var(\hat{\lambda})$: $Var(\hat{\lambda}) = Var\left(\frac{N}{t}\right) = \frac{1}{t^2} Var(N) = \frac{\lambda t}{t^2}$

$$= \frac{\lambda}{t}$$

Aside: Variance of ax
for constant a

$$\mathrm{Var}(ax) = A\left((ax)^2\right) - \left(E(ax)\right)^2$$

$$= E(a^2 x^2) - \left(E(ax)\right)^2$$

$$= a^2 E(x^2) - (a\bar{x})^2$$

$$= a^2 E(x^2) - a^2 \bar{x}^2$$

$$= a^2 \left(E(x^2) - \bar{x}^2\right)$$

$$= a^2 \,\mathrm{Var}(x)$$

Our estimator $\hat{\lambda} = \frac{N}{t}$

is unbiased and has

"standard error" $\sqrt{\frac{\lambda}{t}} = \frac{\sqrt{\lambda}}{\sqrt{t}}$

New twist: k cells

Estimator: $\hat{\lambda} = \frac{N_\Sigma}{kt}$

$$\sigma_{\hat{\lambda}} = \frac{\sqrt{\lambda}}{\sqrt{kt}} = \frac{\sqrt{\lambda/t}}{\sqrt{k}} \approx \frac{\sqrt{\hat{\lambda}/t}}{\sqrt{k}} = \frac{\sqrt{\frac{N_\Sigma/kt}{t}}}{\sqrt{k}}$$

$$= \frac{\sqrt{\frac{N_\Sigma}{kt^2}}}{\sqrt{k}} = \frac{\sqrt{N_\Sigma}}{\sqrt{k^2t^2}} = \frac{\sqrt{N_\Sigma}}{kt}$$

Steps for power analysis:

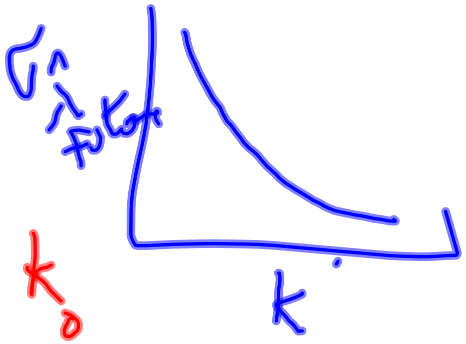
1) Approximate $\hat{\lambda}$

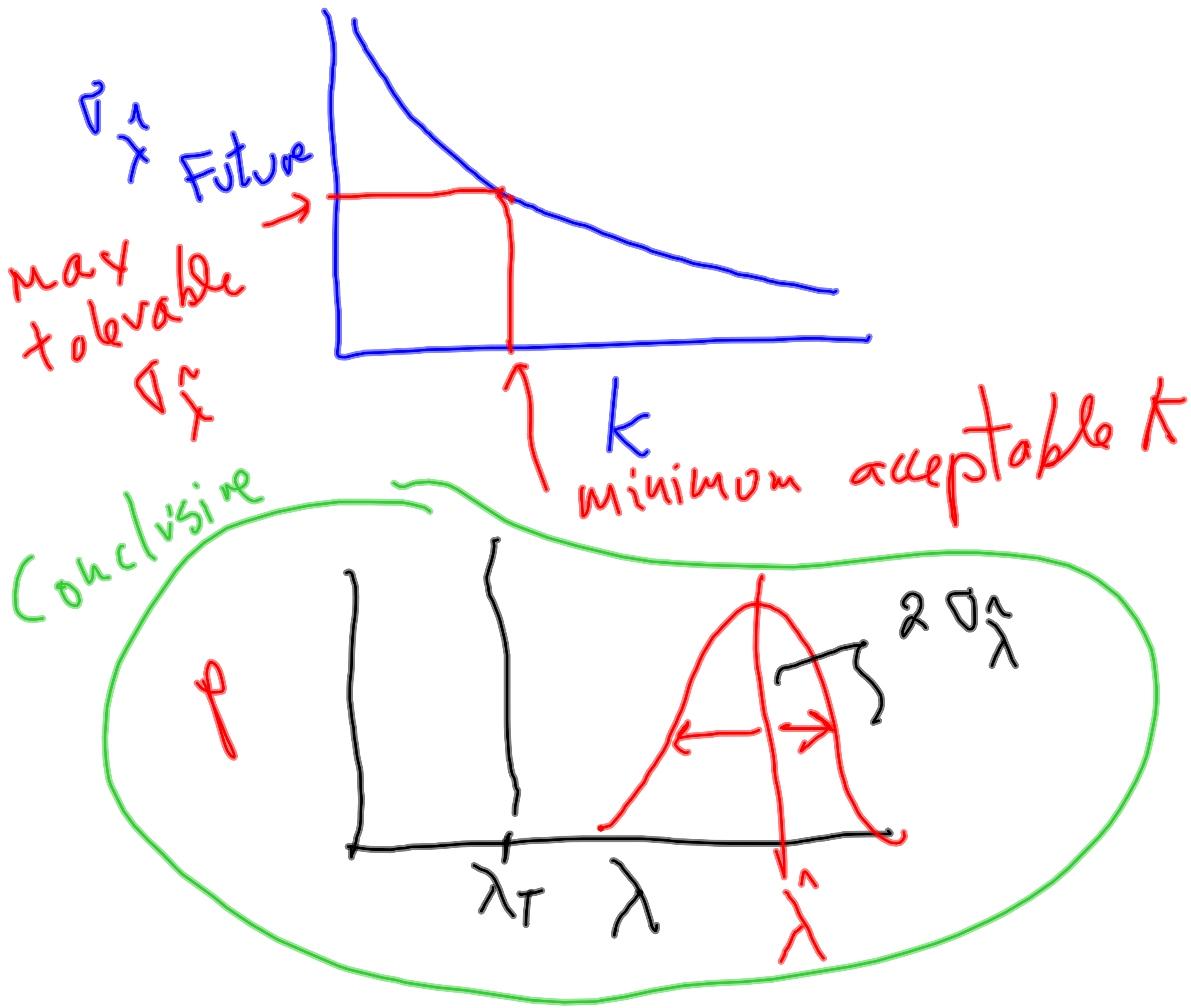
pilot data: $\hat{\lambda} = \dfrac{N_{\varepsilon_0}}{K_0 t}$

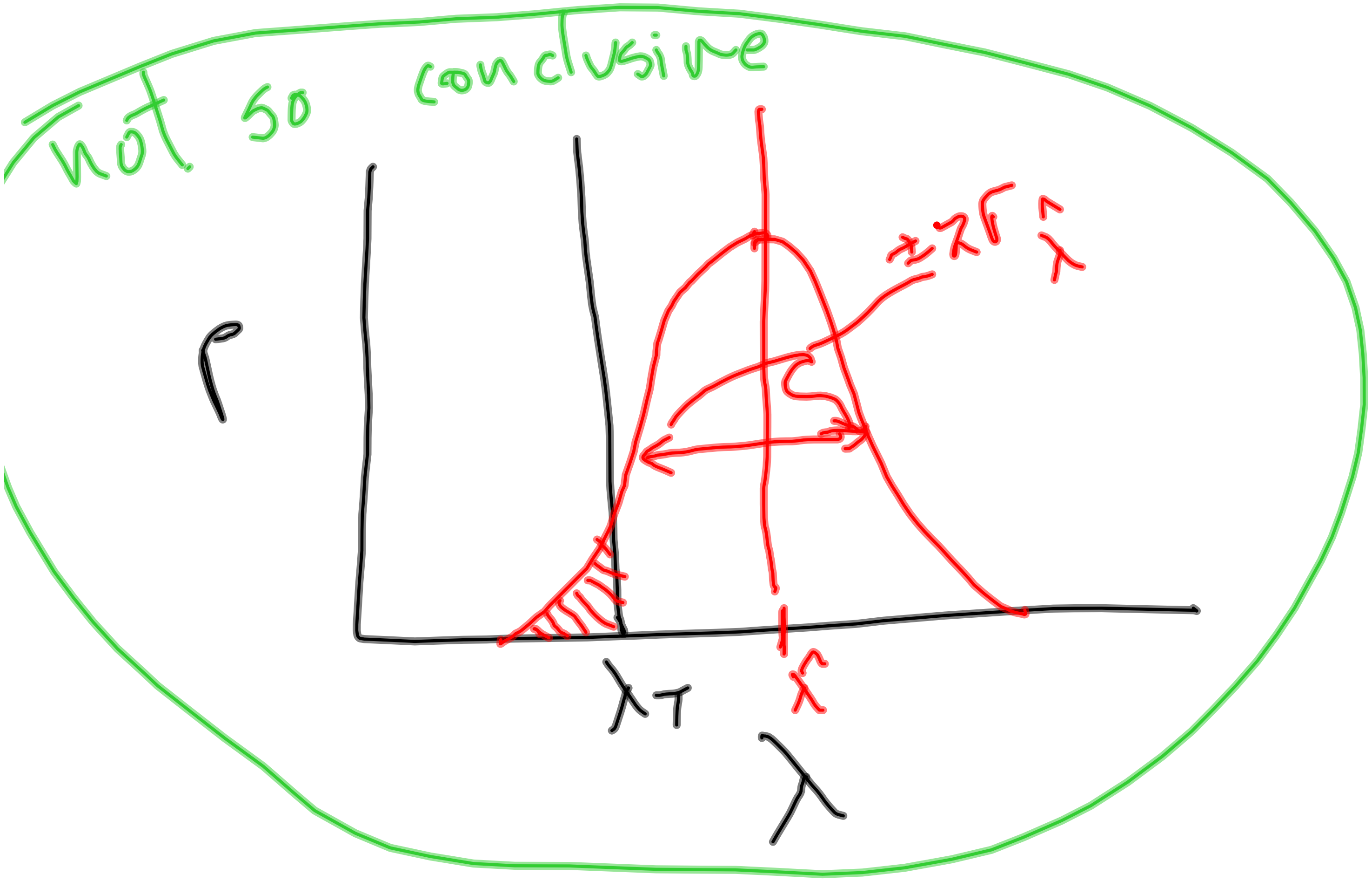
2) Prospective design analysis using K (not necessarily K_0)

$$\sigma_{\hat{\lambda}_{future}} = \frac{\sqrt{\hat{\lambda}/t}}{\sqrt{K_1}}$$

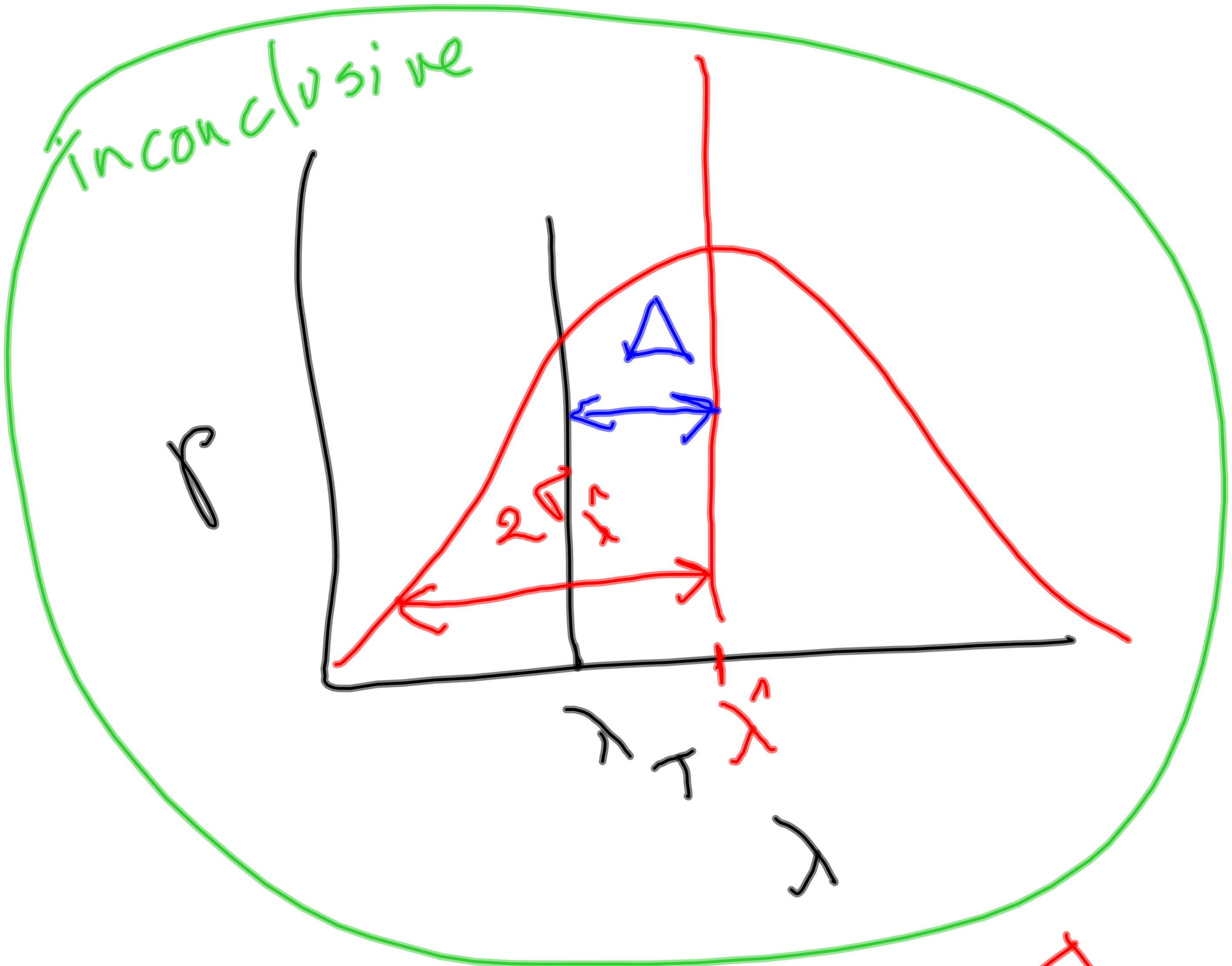
not K_0




$\sigma_{\hat{\lambda}}$ Future
max tolerable $\sigma_{\hat{\lambda}}$
k
minimum acceptable k
Conclusive
p
$2\sigma_{\hat{\lambda}}$
λ_T
λ
$\hat{\lambda}$



not so conclusive
ρ
±2σ
λ̂
λ_T
λ̂
λ



Want $2\sigma_{\hat{\lambda}} < \Delta$

$$\sigma_{\hat{\lambda}} < \frac{\Delta}{2}$$

Ryan's goal is to have 750 ± 50 spores in the treatment

The operational question is: what volume to transfer from the source tank?

$$750 \pm 50 = (\text{Volume transfered}) \times \lambda$$

$$V_{tr} = \frac{750 \pm 50}{\lambda}$$

not known exactly
estimated as $\hat{\lambda}$
subject $\sigma_{\hat{\lambda}}$

$$N_{tr} = V_{tr}\,\lambda$$

$$Var(N_{tr}) = V_{tr}^{2}\,Var(\lambda)$$

$$\sigma_{N_{tr}} = V_{tr}\,\sigma_{\hat{\lambda}}$$

25

$$\sigma_{\hat{\lambda}_{max}} \doteq \frac{\sigma_{N_{tr\,max}}}{V_{tr}}$$

how big does k need to be, so that $\sigma_{\hat{\lambda}} \leq \frac{25}{V_{tr}}$